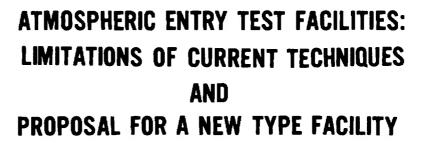
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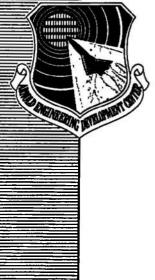
J. Lukasiewicz Virginia Polytechnic Institute

January 1969

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ATMOSPHERIC ENTRY TEST FACILITIES: LIMITATIONS OF CURRENT TECHNIQUES AND PROPOSAL FOR A NEW TYPE FACILITY

J. Lukasiewicz, Virginia Polytechnic Institute

Arnold Engineering Development Center Air Force Systems Command Arnold Air Force Station, Tennessee

Please substitute these Tables III and V for those on pages 17 and 26 in subject report.

TABLE III PERFORMANCE OF MULTISTAGE ROCKET PROPULSION SYSTEMS IN THE 25- to 35-KFT/SEC REGIME WITH Λ = 0.85, X_0 = 10 KFT, r = 100 IN./SEC, I = 200 SEC, AND γ = 0.06 LB/IN.³

V _o , kft/sec	N	WĻ, lb	\mathtt{D}_1 , in.
25	3 .	10	41.6
30	5	4	43.8
35 ·	5	1	47.8

TABLE V
SINGLE-STAGE ROCKET PERFORMANCE

X, kft	5				t 5 10			
D, in.	24		48		2	24		48
r, in./sec	20	50	20	50	20	50	20	50
W _L , Ib	65	162	260	650	130	325	520	1300
W, Ib	910	2275	3640	9100	1820	4550	7280	18, 200

In all cases: R = 4.73, Λ = 0.85, α = 14, γ = 0.06 lb/in.³, Q = 0.46, I = 200 sec, V = 10 kft/sec or I = 300 sec, V = 15 kft/sec

ERRATA

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Please substitute Fig. 9 printed on the reverse side of this page for Fig. 9 on page 24 in subject report. The curve for the Unaugmented Gun was in error by one log cycle on the $W_{\rm L}$ coordinate.

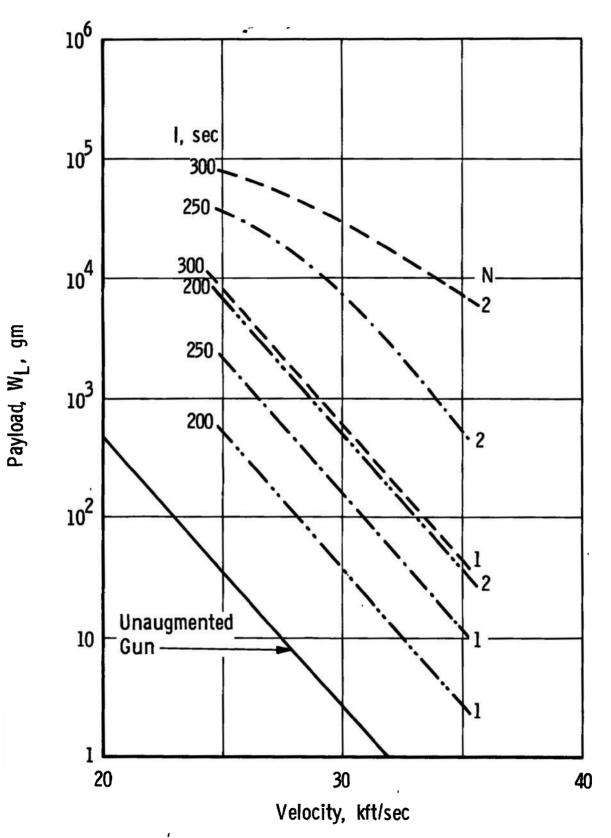


Fig. 9 Performance of Rocket-Augmented Guns

ATMOSPHERIC ENTRY TEST FACILITIES LIMITATIONS OF CURRENT TECHNIQUES AND PROPOSAL FOR A NEW TYPE FACILITY

J. Lukasiewicz* Virginia Polytechnic Institute

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^{*}Consultant, ARO, Inc.

FOREWORD

The work reported herein was sponsored by Headquarters, Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), under Program Element 65401F.

The research was done by Dr. J. Lukasiewicz, Virginia Polytechnic Institute, Blacksburg, Virginia, a Consultant to ARO, Inc. (a subsidiary of Sverdrup & Parcel and Associates, Inc.) contract operator of AEDC under Contract F40600-69-C-0001. The work was started when Dr. Lukasiewicz was Chief, von Karman Gas Dynamics Facility, ARO, Inc. The manuscript was submitted for publication on September 27, 1968.

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ABSTRACT

For many years, considerable efforts have been underway to develop aerodynamic test facilities in which atmospheric entry phenomena could be adequately investigated. The techniques under development comprise wind tunnels of various types, aeroballistic ranges, and counterflow facilities. Theoretical and practical limitations of these facility types are examined in terms of velocity-altitude duplication, and it is concluded that they cannot be expected to provide the desired low altitude, orbital to escape velocity capability with sufficiently large models. suggested that this might be achieved by application of rocket propulsion to aeroballistic-range-type testing. The proposed system for launching of models consists of a multistage rocket booster travelling inside a straight tube evacuated to a low pressure. On attainment of the desired velocity, the model is permitted to proceed into the test range which consists of a variable pressure tank instrumented for aerophysical, erosion, impact, stability, and drag studies. It is estimated that model/ sabot packages weighing from ten to one pound could be launched at speeds from 25 to 35 kft/sec, respectively. The proposed facility is not subject, in the range of interest, to any basic physical limitations, but only to economic constraints. A facility in which the first stage of propulsion is provided by a gun is examined and shown to offer no advantage. Application of the proposed test technique to lower hypersonic speeds is discussed with particular reference to duplication of high Reynolds numbers and to hypersonic ramjet testing.

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A	Rocket motor cross-sectional area	
A*	Rocket nozzle throat cross-sectional area	
a	Acceleration (in g-units)	
a¹	Minimum acceleration	
a"	Maximum acceleration	
С	= $F/(gI^2W)$, Eq. (I-18)	

C_F Thrust coefficient, Eq. (I-27)

D Rocket motor diameter

F Thrust

g Gravity acceleration

I Specific impulse

i Stage number (i = 1, 2, ... N)

N Number of propulsive stages

p_c Rocket chamber pressure

 Q_i See Eq. (I-18) Q_0 See Eq. (I-19)

R Rocket weight (or mass) ratio, = $W_i/(W_i - W_{pi})$

R_O Effective mass ratio, Eq. (I-13)

r Equivalent propellant burning velocity

t Time

V Rocket velocity (or final velocity)

V_O Final velocity

V_S Initial rocket velocity

v = V/(gI)

v_O Final velocity, Eq. (I-12)

W Rocket weight

We Empty rocket (no propellant) weight

 W_L Payload weight (= W_{N+1})

W_p Propellant weight

W_p Propellant weight flow rate

X Distance

X₀ Total distance required to reach final velocity

 α Stage payload ratio, Eq. (I-8)

γ Specific weight of propellant

Δ Increment

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 ϵ = A/A* = nozzle area expansion ratio Λ Motor propellant fraction $\Lambda_{\mathbf{m}}$ Minimum permissible motor propellant fraction

SECTION I

For many years, considerable efforts have been underway to develop aerodynamic test facilities in which atmospheric entry phenomena could be adequately investigated. The techniques under development comprise wind tunnels of various types, aeroballistic ranges, and counterflow facilities (which combine wind tunnel and range techniques). Using these approaches, it is increasingly difficult (see, e.g., Refs. 1, 2, 3, and 4) to attain desired performance with respect to velocity, ambient density, and model size. Wind tunnels operating in the 25- to 35-kft/sec range at densities of major interest (altitudes below 200 kft) have yet to be developed, whereas guns of significant size are limited to about 25 kft/sec. This situation results from practical constraints related to energy requirements, energy losses, containment of high pressures, permissible model accelerations, etc. In summary, it is apparent that conventional techniques may be reaching a performance plateau short of attaining the desirable goals.

Free, atmospheric flight with full-scale or subscale models is at present the only other source of data. The usefulness of this technique, however, is severely limited because of excessive costs and difficulties of acquiring reliable measurements.

Bearing these circumstances in mind, a new type of a large, ground-based atmospheric entry test facility is here proposed. It is, essentially, a small-scale (compared with full-scale flight hardware) free-flight facility of the aeroballistic range type in which the model is accelerated to the desired speed by a multistage rocket booster (rather than with a gun). The construction of such a facility would constitute a major undertaking, although it would not necessarily be more expensive than such past aerodynamic facility projects as the post-war Unitary Wind Tunnel Program. Just as the Unitary Wind Tunnels, the proposed facility should be conceived as a national resource, serving the needs of the Government and the private sector.

SECTION II LIMITATIONS OF EXISTING ATMOSPHERIC ENTRY TEST FACILITIES

As already indicated, the need for development of a new type of atmospheric entry test facility stems from basic limitations of techniques thus far used: wind tunnels, aeroballistic ranges, and their combinations.

2.1 WIND TUNNELS

Wind tunnels, in which the test model is stationary in the laboratory frame of reference, usually rely on steady, isentropic expansion of the working fluid to the desired ambient conditions. Although in this manner they can provide aerodynamic duplication in terms of such (ideal gas) similarity parameters as Mach and Reynolds numbers, they are unable to reproduce at the same time the velocity and the ambient density and temperature (pressure). The latter requirements, which amount to duplication (rather than aerodynamic simulation) of the environment, arise at velocities at which significant thermochemical-kinetic (TCK) effects (such as dissociation, ionization, nonequilibrium) appear in the model flow field, and phenomena related to thermal properties of model materials (such as ablation) are present.

Limitations of steady, isentropic flow expansion-type wind tunnels are given in Fig. 1. This is a Mollier chart for air (Ref. 5), on which atmospheric conditions at altitudes from 150 to 350 kft (entropy s/R range from 30 to 39) are indicated. At the top of the chart, 10,000-and 5000-atm isobars are shown as corresponding to limiting stagnation pressures which may be in practice contained in the reservoir region of a wind tunnel. A velocity scale, included with the enthalpy H/R (°K) ordinate, indicates that the speeds attainable through isentropic expansion to ambient atmospheric conditions are limited to values between 16 kft/sec at 150 kft and 20 kft/sec at 250 kft. Moreover, comparison of the two reservoir isobars shows that a large increase in pressure results in only small augmentation of velocity.

It is thus evident that, based on pressure or structural limitations alone, velocities attainable by steady, isentropic expansion to ambient conditions below 200-kft altitude are smaller than about 18 kft/sec and fall short of the atmospheric entry speeds of major interest, which span the range from about 23 kft/sec (ballistic missiles) to 36 kft/sec (lunar mission entry).

¹Assumed equivalent to total enthalpy (i. e., neglecting static enthalpy after expansion to free-stream conditions).

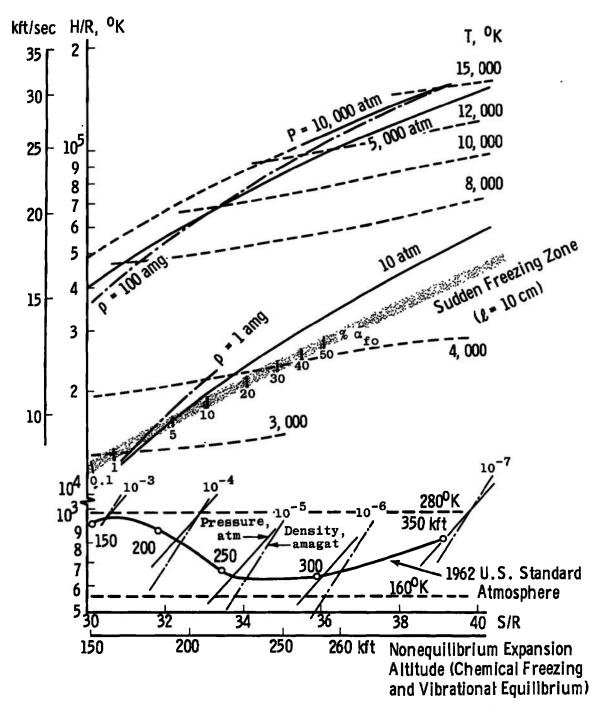


Fig. 1 Mollier Diagram for Air Showing Reservoir Conditions in Relation to Flow Velocity,
Duplication of Atmospheric Conditions, and Amount of Frozen Atomic Oxygen

An additional limitation of the steady, isentropic expansion technique, caused by the relaxation processes in the nozzle, is also, indicated in Fig. 1. Because of the finite relaxation rates, equilibrium air composition is not maintained in the course of expansion, and composition of the test section flow differs from equilibrium air composition. It has been shown² that, as the expansion reaches the "sudden freezing zone" (Fig. 1), the flow composition becomes virtually "frozen." the actual composition being a function of the entropy only (to a good approximation). Since altitude conditions are also uniquely defined by entropy, there is a direct relationship between the flow composition and altitude attainable by isentropic expansion (as originally pointed out in Ref. 1). In the thermodynamic range of interest, the atomic (dissociated) oxygen is the most important species. In Fig. 1, the percent fraction (α_{fo}) of atomic (frozen) to total oxygen is indicated for the case of a largescale facility. 3 It is evident that at velocities above 20 kft/sec, $\alpha_{\rm fo} > 10$ percent, and the test section flow composition differs substantially from that of the atmosphere; the correspondence between the test section conditions and the free-flight conditions becomes ambiguous. In Fig. 1, the bottom scale gives the altitude based upon matching of the freestream density and frozen velocity of sound in the test section with the atmospheric values (as suggested and estimated in Ref. 7); this results in a slightly lower altitude at a given entropy but does not significantly change the duplication capability of steady, isentropic expansion. 4

²See Ref. 6 for a comprehensive review of nonequilibrium effects in hypersonic nozzle flows.

³Corresponding to a scaling factor ℓ = 10 cm, with ℓ = r*/tan $\theta/2$, $\theta/2$ = conical nozzle half-angle, A/A* = 1 + $(X/\ell)^2$, A*, r* = nozzle throat area and radius, X = distance from nozzle throat. For ℓ = 10 cm and M ≈ 20, the test section core diameter would be on the order of 10 ft.

⁴Performance superior to that attainable with steady, isentropic expansion is possible under more restrictive conditions of simulation or, theoretically, by other gas-dynamic means. For example, if Mach number duplication were dropped, as may be appropriate for blunt bodies, and only stagnation conditions duplicated, simple shock tubes (Refs. 8 and 9) offer duplication at velocities up to 60 kft/sec and at altitudes below 200 kft (e.g., 40 kft/sec at 100 kft). Theoretically (Refs. 10, 11, and 12), unsteady compression and/or expansion offer performance much superior to that obtainable with the steady, isentropic process; however, because of practical difficulties, thus far it has not been possible to utilize for aerodynamic testing the gas processed by an unsteady wave region (Ref. 13).

The performance attainable with steady, isentropic expansion can be augmented by electromagnetic acceleration. Optimization studies of a crossed-field (Faraday)-type accelerator have been published by Norman and Chmielewski (Ref. 7), and the results, in the velocity-altitude plane, are shown in Fig. 2, for a supply reservoir pressure of 5000 atm, and for various values of the parameter B²L, where B is the applied magnetic field and L is the accelerator length. The accelerator is located in the initial, supersonic portion of a hypersonic nozzle, supplied from, e.g., a high-performance shock tunnel. The optimization involves minimization of the final entropy for a given test section velocity (equivalent to maximization of velocity for a given entropy or altitude).

Since for efficient operation of the accelerator (i.e., for a small entropy rise) a sufficiently large electrical conductivity is required, seeding of air with an easily ionized material is necessary. A seeding fraction of 1/4 percent by mass has been assumed; although a larger value would slightly improve accelerator performance, the resulting contamination would make the interpretation of some results more difficult.

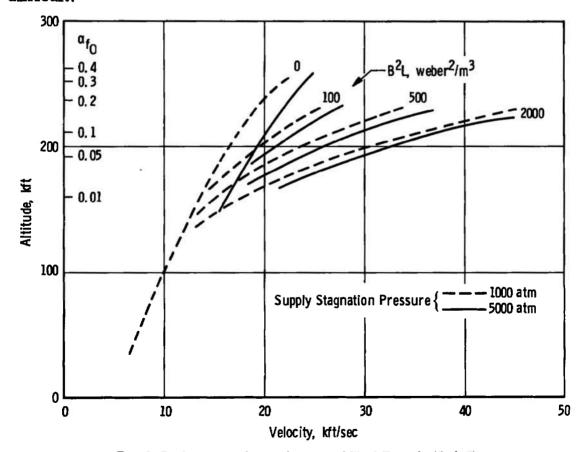


Fig. 2 Performance of MHD-Augmented Wind Tunnels (Ref. 7)

As regards values of the B^2L parameter, the higher ones (500 and 2000 weber $^2/m^3$) may be difficult to attain in practice: they would correspond, for example, to an accelerator length of 5 m and magnetic fields of 10 and 20 weber/ m^2 , respectively, with the electric fields of 100 kv/m and 200 kv/m.

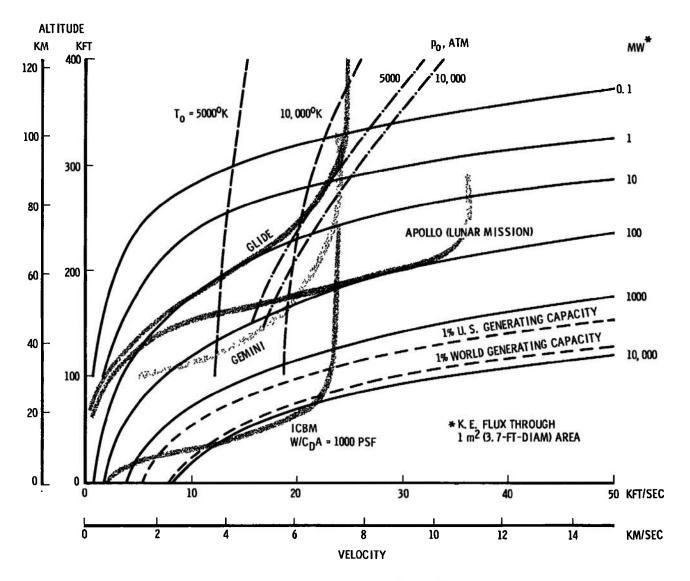
The nozzle expansion nonequilibrium phenomena would not be alleviated by the accelerator; as before, freezing of chemical composition would occur in the nozzle expansion downstream of the accelerator, the frozen atomic oxygen fraction being again uniquely related to the altitude. The values of α_{fo} are indicated on the altitude scale in Fig. 2; the limit duplication lines are drawn for matching of density and velocity of sound, for chemically frozen and vibrationally equilibrated expansion.

Figure 2 shows that, even for the larger values of the parameter B^2L , velocities in excess of 20 kft/sec are unattainable at altitudes below 150 kft. For the more realistic value of 100 weber $^2/m^3$, duplication in this speed range is possibly only above 200-kft altitude.

The second limitation of wind-tunnel-type facilities, although less basic but nevertheless in practice important, relates to the test section energy flux under conditions of interest. Figure 3 describes the situation: duplication of ICBM and lunar flight (Apollo) atmospheric entry requires energy fluxes from 10,000 to 100 megawatts per square meter (3.7-ft diameter) of test section, the larger value exceeding one percent of the world's electrical generating capacity. Since large losses accompany acceleration of the fluid to the desired velocity and recovery of the kinetic energy is not possible, the actual power requirements exceed by a large factor the ideal values. Thus, on account of the energy requirements alone, only short run times, on the order of a millisecond or less, in relatively small test sections could be obtained with the required, extremely high power levels being realized with some form of energy storage, e.g., capacitors or high explosives.

Summing up the limitations of the wind tunnel technique, it is apparent that the theoretically attainable performance is limited to altitudes in excess of from 150 to 200 kft and that, because of extremely high energy requirements and attendant costs, test section size and flow duration are in practice restricted to modest values. ⁵

⁵Other factors, such as viscous effects and radiative losses, also limit flow duration.



~

Fig. 3 Test Section Kinetic Energy Flux

2.2 AEROBALLISTIC RANGES

The other basic test technique, which constitutes essentially the miniaturization of free, atmospheric flight, is the aeroballistic range. As regards atmospheric entry testing, its main advantages are two-fold: (1) production of relative velocity is decoupled from provision of ambient conditions, which can be therefore easily controlled (usually by pressure level in the range tank) and (2) high velocities can be attained with small models launched from two-stage, light-gas guns. Compared to a wind tunnel, the main disadvantage of the range technique is the difficulty of making many types of model measurements; on the other hand, the technique is particularly well suited to observation of some phenomena, such as wakes.

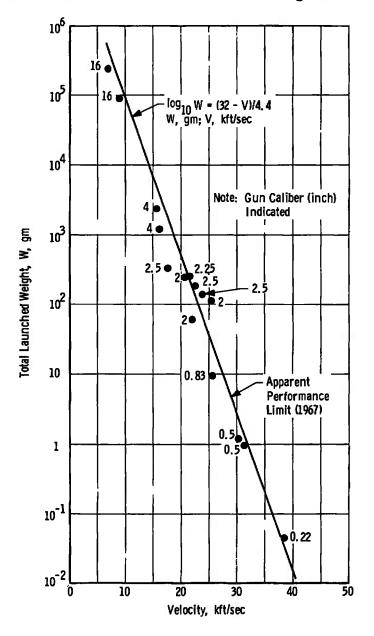
At present, the limitations of the aeroballistic range technique rest with the gun; they have been discussed in detail in Ref. 2 in terms of both theoretical and actual performance.

The maximum launch velocity available with current two-stage, light-gas guns is shown in Fig. 4a (see also Ref. 2) in terms of the total launched weight; the calibers of the "record guns" are indicated. Also included are two points for a 16-in., high performance, single-stage powder gun (Ref. 17). In Fig. 4b the corresponding gun calibermaximum velocity correlation is shown. At velocities larger than 25 kft/sec, the launched weight (of which the model constitutes only a fraction) is smaller than 100 gm; moreover, it decreases tenfold for every 4400-ft/sec velocity increase. The velocity of 35 kft/sec can be attained only with projectiles weighing less than about 0.2 gm.

Although, ideally, scaling of a gun to any caliber (and to any launched weight) is straightforward, Fig. 4 clearly shows that this is not borne out by practice. The failure is due to difficulties of containing very high pressures in large-diameter gun components. On the theoretical side, Ref. 2 indicates that, even for very small models, light-gas guns of conventional design do not offer much potential above the 35-kft/sec velocity level.

One other limitation of the gun launching is the high base pressure (or acceleration) load to which it subjects models. With current guns, base pressures in excess of 50 kpsi are experienced at velocities above 20 kft/sec. The resulting stresses allow only simple model shapes to

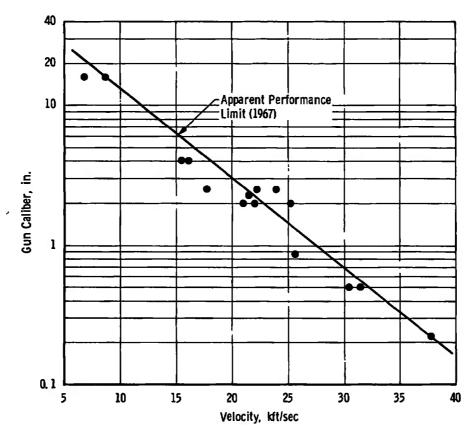
be launched, and thus far, prohibit inclusion of on-board telemetry for transmission of measurements from models in flight. ⁶



a. Velocity versus Weight

Fig. 4 Maximum Launch Velocities

⁶Telemetry with gun-launched models has been successfully achieved at lower velocities (Ref. 14). Further progress may be expected from application of modern microminiaturization-integrated circuit techniques; however, difficulties might still be present due to acceleration loading of sensors.



b. Velocity versus Gun Caliber
Fig. 4 Concluded

Thus, current limitations of the aeroballistic range technique concern the size of models that can be launched at atmospheric entry speeds and the base pressure loading which they have to experience in the process. Scaling up of the launcher caliber by a large factor would remove the former limitation but, because of structural difficulties, appears impractical.

2.3 COUNTERFLOW FACILITIES

Speeds higher than available with either of the two techniques discussed above can be achieved in counterflow-type facilities, in which models are gunlaunched counter current to tunnel test section flow (Ref. 15). However, except for velocity performance, not only the

⁷For a given launch cycle, model weight varies as the cube of the gun caliber while the base pressure remains constant, the time scale being proportional to the caliber (Ref. 2).

limitations of the two constituent techniques are present, but also the time (or distance) of test is curtailed (as compared with aeroballistic ranges) because of the inability of maintaining acceptable quality, high velocity flow over long distances and long times.

SECTION III GENERAL DESCRIPTION OF THE PROPOSED FACILITY

As mentioned in Section I, the proposed facility is essentially an aeroballistic range in which rocket propulsion is used to accelerate the model to the desired velocity.

The envisaged launch system consists of a multistage rocket vehicle (booster in aerospace terminology) travelling inside a straight, cylindrical tube (or a series of tubes of decreasing number or diameter) evacuated to a low pressure. The purpose of this confinement of the booster is twofold: it provides guidance and eliminates aerodynamic drag. These, in fact, are the essential factors which distinguish the proposed system from the free-flight testing in the atmosphere using conventional rocket propulsion. The absence of atmospheric resistance is necessary to reduce the distances required and to eliminate aerodynamic heating, and the built-in guidance simplifies the booster stages which are expendable. On attainment of the desired velocity, the model or payload is permitted to proceed into the test range, and the booster stages are decelerated and, most likely, destroyed, each on completion of its propulsive run. The test range consists of a suitably sized vacuum tank that is instrumented for aerophysical, stability, drag, erosion, and impact studies and equipped to provide an atmosphere of desired pressure and composition (as required for atmospheric entry simulation). Within the test tank, the model is allowed to fly freely, or its mechanical guidance can be continued with small aerodynamic interference by, e.g., three slender rails supported radially along the length of the test tank. The latter, "captive" mode of operation may be particularly suitable for nose heating, erosion, and impact testing.

In the evaluation of performance attainable with multistage rocket systems, we shall focus our attention on the 25- to 35-kft/sec velocity range. Although for some interplanetary missions much higher entry speeds into the earth's atmosphere must be envisaged, these are the velocities of main interest at the current stage of space flight development. They are, specifically, suborbital and orbital velocities applicable to the delivery of ballistic and orbital payloads, i.e., from about 23 to 26 kft/sec, and escape velocity from the earth or re-entry velocity from a lunar mission, of about 36 kft/sec. It is reasonable to expect

that over the next decade atmospheric entry velocities in these two regions will account for the large majority of space flights.

As regards altitude simulation, this presents no difficulties in the facility here proposed, the atmospheric conditions being obtained by control of pressure inside the range tank. If more sophistication were desired, the composition as well as temperature and pressure of the tank atmosphere could be controlled.

SECTION IV PERFORMANCE OF MULTISTAGE ROCKET PROPULSION SYSTEMS

4.1 PURE ROCK ET PROPULSION

Multistage rocket performance is basically given by Eq. (I-19), see the Appendix, in terms of parameter Q_0 , and other related quantities. Q_0 can be expressed as

$$Q_o = \frac{\pi \gamma r D_1^2 X_o}{4g \text{ IW}_L \alpha^N}$$

and therefore, for a given number of stages N, final velocity V_0 , motor propellant fraction Λ , and propellant properties I and γ , we have

$$Q_o \propto r D_i^2 X_o / W_L$$

where

r = equivalent propellant burning velocity

D₁ = diameter of first stage rocket motor

 X_o = total distance required to reach final velocity V_o (=total launcher length)

WL = payload weight

The variation of the lumped parameter r $D_1^2 X_0/W_L$ in terms of Λ for the specified values of V_0 and N is given in Fig. 5. Values of I=200 sec and $\gamma=0.06$ lb/in. 3 have been taken for the specific impulse and the specific propellant weight. From the curves in Fig. 5 the performance available for a given value of the lumped parameter and the tradeoffs possible can be quickly determined. The powerful effect of the motor propellant fraction, Λ , particularly when smaller numbers of stages are used, is evident.

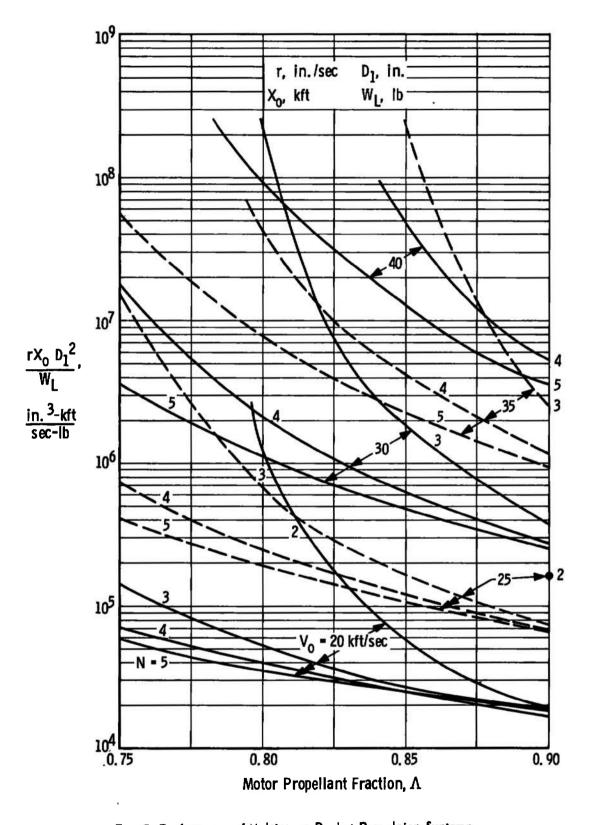


Fig. 5 Performance of Multistage Rocket Propulsion Systems

As shown in the Appendix (Eq. (I-9)), there is a minimum permissible motor propellant fraction, $\Lambda_{\mathbf{m}}$, for which the payload vanishes. For the conditions assumed (I = 200 sec), the minimum possible values of Λ are given in Table I. It is immediately apparent that, for the velocities of interest and for realistic motor propellant fraction values (< 0.9), multistaging is necessary.

TABLE I MINIMUM VALUES OF MOTOR PROPELLANT FRACTION, Λ_m (I = 200 sec)

\/ \kft\sec	· N						
V _o , kft/sec	1	2	3	4	5		
10	0.79	0. 54	0. 40	0.32	0. 27		
15	0.90	0. 69	0. 54	0. 44	0.37		
20	0. 95	0.79	0. 65	0.54	0. 46		
25	6. 98	θ. 86	0. 73	0. 62	0. 54		
30	0. 99	0.90	0.79	0. 69	0.61		
35	> 0.99	0.93	0.84	0.74	0.66		
40	> 0. 99	0. 96	0.87	0. 79	0.71		

 $---\Lambda_{\rm m}$ = 0.85 boundary

In order to interpret the results shown in Fig. 5, meaningful assumptions must be made concerning rocket characteristics: the motor propellant fraction, Λ , the equivalent burning velocity, \mathbf{r} , the specific impulse, \mathbf{I} , and the specific propellant weight, γ .

It is not our purpose here to examine the current and probable future limitations on the magnitude of Λ and \mathbf{r} , limitations that can be determined only after detailed design studies of the proposed system. However, some indication of what might be possible can be gained from information at hand.

Light, fast burning rocket motors have been developed for propulsion of rocket sleds. Characteristics of one such motor, developed in 1966 by the Atlantic Research Corporation under contract to Sandia Corporation (Ref. 16), are as follows: propellant fraction $\Lambda = 0.715$, average specific impulse I = 237 sec, nozzle expansion ratio $\epsilon = 3$, equivalent propellant burning velocity (based on outside nozzle diameter of 5.5 in.; maximum

motor case diameter = 5.2 in.) r = 51 in./sec, action time 1.08 sec, average pressure $p_c = 1920$ psi, average thrust coefficient $C_F = 1.29$, max acceleration load sustained in test = 315 g.

With this motor, shown in Fig. 6, high specific impulse and equivalent burning velocity have been achieved, as well as high acceleration loadings. As regards propellant fraction, it has been suggested (Ref. 16) that it could be increased by the use of higher strength/density ratio case materials, such as maraging steel instead of aluminum.

Other multi-stage designs of high performance rocket motors (solid and liquid) are mentioned in Ref. 17; a motor propellant fraction from 0.8 to 0.85 was estimated for these designs.

In view of the above considerations, a value of 0.85 could be taken as a realistic goal for the motor propellant fraction Λ , and values of r of perhaps 50 in./sec and higher could be envisaged.

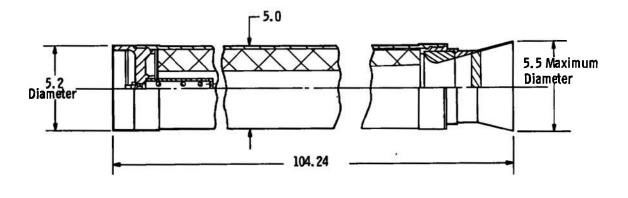


Fig. 6 Atlantic Research Corporation Fast Burning Rocket Motor (Ref. 16)

All Dimensions in Inches

As stated, a rather modest value of 200 sec has been taken for the specific impulse, with a typical solid-propellant specific weight of 0.06 lb/in.^3 . It was considered prudent to assume a low specific impulse in view of the small nozzle expansion ratio which is likely to be used in order to maximize thrust (see Eq. (I-27)) and thus minimize the total required launcher length, X_0 . As regards stresses to which

the payload is subjected during acceleration, they are given in terms of thrust pressure by (see Appendix, Eqs. (I-1) and (I-21)) $F/A = r\gamma I$ and, for the above-indicated values of r, γ , and I, would not exceed 1200 psi, i.e., would be an order of magnitude smaller than base pressures encountered in high velocity guns.

In order to get some indication of the ranges of parameters involved, several specific cases are listed in Table II for a 1-lb payload. The interplay of parameters r, X_0 , and D_1 for a given velocity V_0 and number of stages N is apparent. The powerful influence of Λ in the 40-kft/sec case is demonstrated.

In Table III the choice is narrowed to three specific cases, all for $X_0 = 10$ kft and r = 100 in./sec (or $X_0 = 20$ kft and r = 50 in./sec. With the first stage launch tube smaller than 4 ft in diameter, payloads from 10 to 1 lb can be launched at velocities of 25 to 35 kft/sec, respectively.

We shall focus our attention on the system giving the higher of the two velocities of particular interest, i.e., $35 \, \mathrm{kft/sec}$; more detailed data for such a system are collected in Table IV. The total length of the system, for $r = 100 \, \mathrm{in./sec}$, is 10 kft; the same performance would be obtained with a 20-kft system and $r = 50 \, \mathrm{in./sec}$, a more realistic value. The maximum accelerations experienced would be 3310 and 1655 g, respectively. The first stage launch tube consists of a 4-ft-diam, 340-ft-long tube (for $r = 100 \, \mathrm{in./sec}$), the second and higher stage launch tubes being progressively smaller in diameter and longer; the total tube volume is only about 9000 ft³.

It is of interest to examine the performance attained with three and four stages only: W_L = 20.6 lb at 21 kft/sec and W_L = 4.54 lb at 28 kft/sec. These are very worthwhile launch capabilities at sub-escape velocities.

4.2 GUN-AUGMENTED ROCKET PROPULSION

The obvious complication in the proposed rocket propulsion system is the mechanics of staging. It is therefore of interest to estimate performance attainable with a gun used as the first stage of propulsion and to determine the extent of rocket propulsion system required in conjunction with the gun. The tradeoff could involve elimination of staging or reduction in the number of stages, at the penalty of high acceleration loading during the gun launch.

TABLE II DESIGN CHARACTERISTICS OF SELECTED MULTISTAGE ROCKET PROPULSION SYSTEMS WITH I = 200 SEC, W = 1 LB, AND γ = 0.06 LB/IN.3

V _o ,			r.	X _o ,	
kft/sec	N	W ₁ , Ib	r, in./sec	kft	D ₁ , in
Λ = 0.85		•			
20	2	190	20	5	23.6
	2	190	10	5	33.4
	2 2 3 3	71	10	5	23.5
	3	71	10	1	52.5
	3	71	20	1	37.2
	3	71	50	1	23.5
25	3	321	10	5	58.8
	3	321	20	5	41.6
	3 3 3	321	50	2.5	41.6
	3	321	100	2.5	29. 4
	3	321	100	1	41.6
	4	190	20	5	34.7
	4	190	100	1	34.7
30	4	757	10	5	112.5
	4	757	20	5	79.6
	4	757	50	5 5 2.5	50.3
	4	757	100	2.5	50.3
	5	514	10	5	98. 2
	5	514	20	5	69. 4
	5 5 5	514	50	5	43.8
	5	514	100	2.5	43.8
	5	514	50	10	31
35	5	1928	20	10	107.1
	5 5 5	1928	50	10	67.6
	5	1928	100	5	67.6
	5	1928	100	10	47.8
	5	1928	50	20	47.8
40	5	8632	20	20	177
	5	8632	50	20	111.7
	5	8632	100	20	79
Λ = 0.9					
40	5	2466	20	20	94.6
	5	2466	100	20	42.3

TABLE III PERFORMANCE OF MULTISTAGE ROCKET PROPULSION SYSTEMS IN THE 25- TO 35-KF T/SEC REGIME WITH X $_{\rm o}=10$ KFT, r = 100 IN./SEC, I = 200 SEC, AND $\gamma=0.06$ LB/IN. 3

V _o , kft/sec	N	W _L , Ib	D_1 , in.
25	3	10	41.6
30	5	4	43.8
35	5	1	47.8

TABLE IV FIVE-STAGE ROCKET SYSTEM FOR W_L = 1 LB AT 35 KFT/SEC WITH Λ = 0.85, r = 100 in./SEC, X_o = 10 KFT, a'' = 3310 g, t_i = 0.12 SEC, I = 200 SEC, AND γ = 0.06 LB/in.³

i	1	2	3	4	5	Total
V _i , kft/sec	7	14	21	28	35	
x _i , ft	341	1171	2000	2829	3659	10, 000
D _i , in.	47. 8	22. 4	10. 53	4. 95	2.32	
Launch Tube Volume, ft ³	4250	3200	1200	378	108	9, 136
W _i , lb	1928	425	93.5	20. 6	4. 54	

It should be noted in passing that such systems of propulsion have been proposed before mainly for the purpose of augmentation of gun performance.

Early, unsuccessful and abandoned efforts (e.g., Refs. 18, 19, and 20), were concerned with the so-called travelling charge gun, an arrangement in which some propellant was carried on the base of the projectile. Only very small and erratic (hence useless for artillery applications) increases in muzzle velocity were observed using this technique. No throat or nozzle was provided to accelerate the propellant gases and to maintain combustion pressure, which was progressively decreasing because of the rarefaction accompanying the projectile acceleration. Clearly, only a small specific impulse could have been realized with this primitive technique.

More recently, the use of gun-launched rockets was being considered as a means of increasing the range of guns (Ref. 21) and for atmospheric sounding, flight testing, and satellite launching (Ref. 17) applications. Except for some aspects of the design of rocket motors, these systems, inasmuch as they involve atmospheric flight, are not comparable to the system here considered.

In order to determine optimum performance of gun-rocket systems, the maximum gun performance as indicated by the 1967 limit in Fig. 4a was assumed, and, for a given number of stages, specific impulse, and motor propellant fraction, the maximum payload was calculated. Typical results leading to such optimization are shown in Fig. 7. For this particular case (Λ = 0.85, I = 200 sec, N = 3, where N equals the number of rocket propulsion stages), a gun launch at about 5 kft/sec results in maximum payload for a final velocity of 35 kft/sec; however, for lower velocities, maximum payload is attained at zero velocity gun launch, i.e., the result is trivial and indicates that no advantage can accrue through gun application. This situation, or one in which payload maximum occurs at a very low gun launch velocity (< 5 kft/sec), is generally found for $N \ge 2$. Therefore, in order to be meaningful, the results shown in subsequent figures are in all cases for a launch velocity not smaller than 5 kft/sec. Actually, the gun-launched weight at 5 kft/sec, as given by the limit line used here (Fig. 4a), exceeds by some 75 percent the actual performance of a 75-caliber, 16-in. gun (1700 lb, Ref. 17), and therefore the quoted payload estimates are optimistic for N > 2 cases.

The results are given, for I < 200, 250, and 300 sec, in Figs. 8a, b, and c and summarized, for N = 1 and 2, in Fig. 9; in all cases $\Lambda = 0.85$ has been assumed.

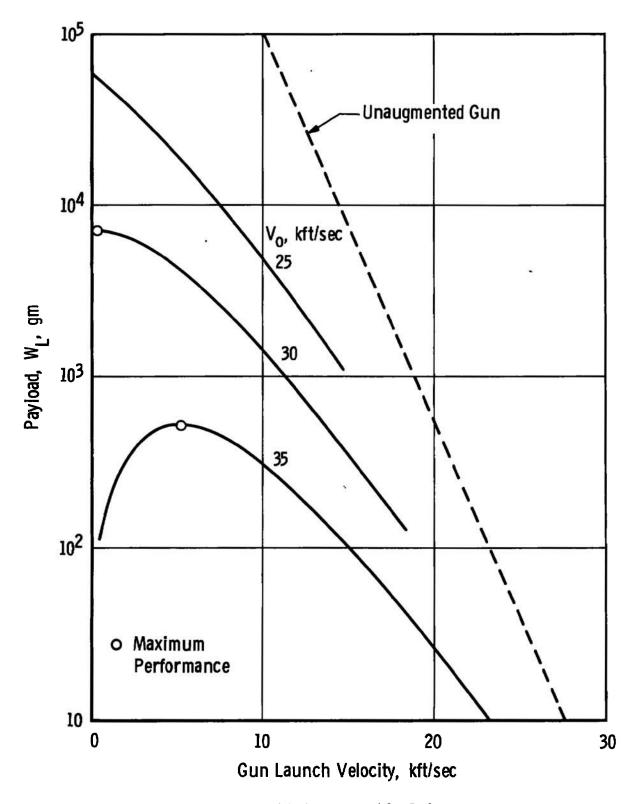


Fig. 7 Optimization of Rocket-Augmented Gun Performance

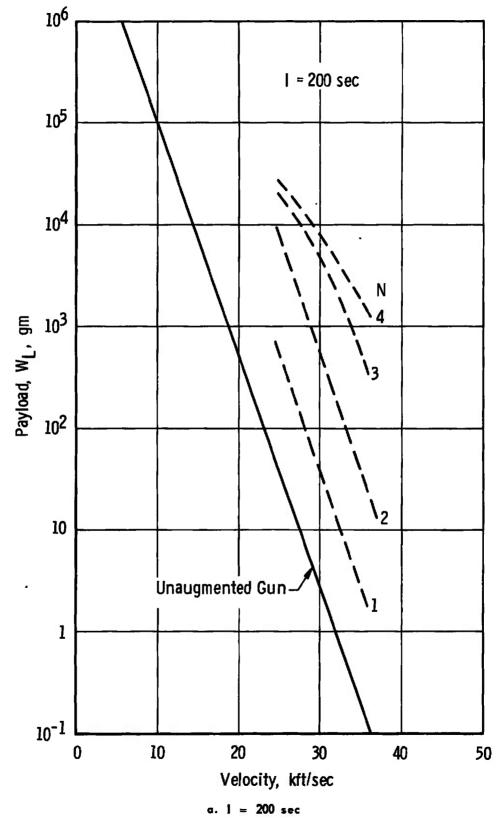


Fig. 8 Performance of Rocket-Augmented Guns

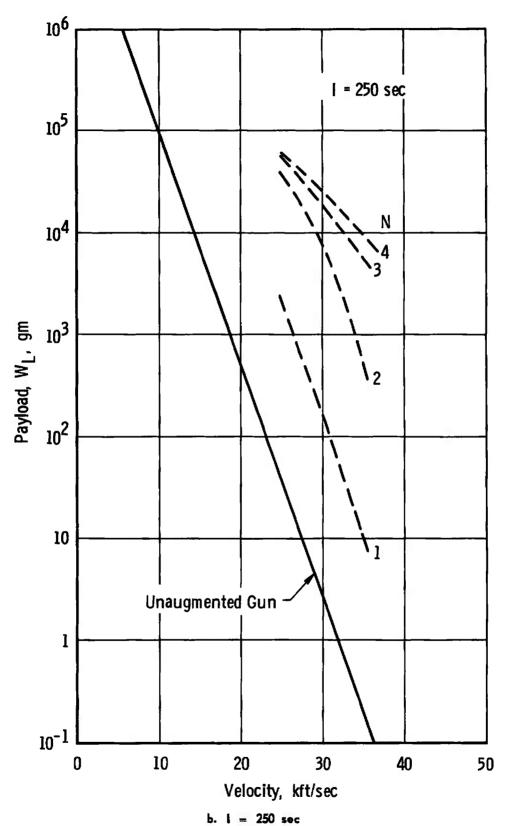
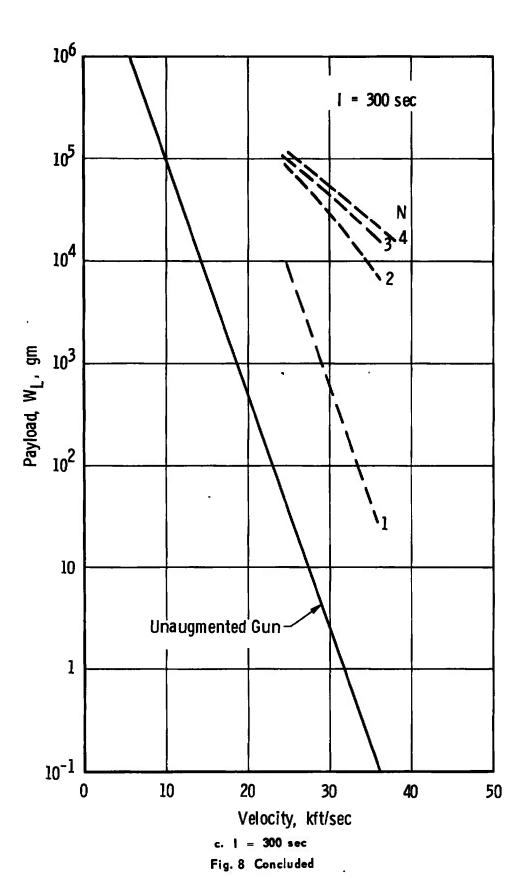


Fig. 8 Continued



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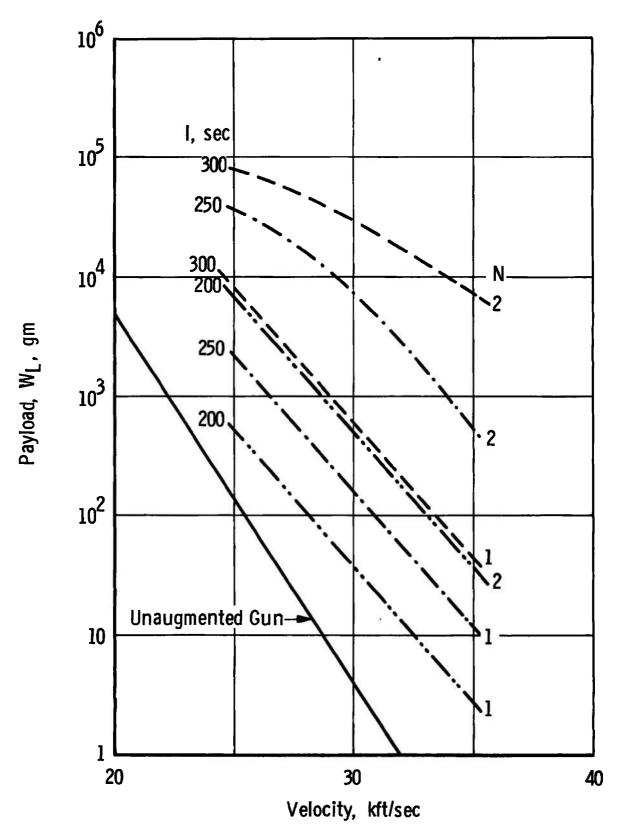


Fig. 9 Performance of Rocket-Augmented Guns

It is immediately apparent from Fig. 9 that, with a single-stage rocket, although a very significant improvement over the unassisted gun can be obtained for the high specific impulse case, at $35 \, \text{kft/sec}$ the available payload is small, on the order of 50 gm. Two stages are required, with I = 250 sec, to increase the payload to a more significant value of 500 gm.

Figure 9 indicates that, at least in the higher end of the velocity spectrum of interest, multistaging is required in conjunction with the gun, which therefore offers no benefits. The 25-kft/sec case might be of more interest: a single stage with I = 250 sec gives a payload of 2000 gm, more than tenfold the weight which can be now launched with an unassisted gun. The optimum launch velocity in this case is 11.9 kft/sec, and the corresponding launch weight is 37 kg. A two-stage gun of about 10-in. caliber (see Fig. 4b) would be required; the length of the rocket run would be about 4700 ft (assuming an equivalent burning velocity r, of 50 in./sec). Assuming a barrel 150 calibers long (125 ft), the in-gun package would have to be subjected to an equivalent, constant base pressure of about 17.5 kpsi or to an actual peak pressure⁸ of some 35 kpsi. The rocket propulsion would result in a thrust pressure of only 750 psi. Thus, with the use of the gun as first stage propulsion, the rocket motor and payload would be subjected to relatively high stresses.

SECTION V PERFORMANCE AT LOWER HYPERSONIC SPEEDS

The proposed type of facility, in which production of velocity is decoupled from provision of ambient conditions, need not be restricted to investigation of atmospheric entry but could also provide useful information in other flight regimes. Applications to lower hypersonic speeds, under conditions unattainable with conventional aeroballistic ranges (because of their small size) or wind tunnels and rocket test sleds (because of low velocity) are indicated in this section.

The lower hypersonic regime (Mach number 10 to 15) may be of interest in terms of duplication of flight Reynolds numbers and heating rates or for air-breathing propulsion. In the lower portions of the

⁸Because of imperfections of the actual launch cycle, a peak pressure about double the ideal, minimum value would be expected (Ref. 2).

ballistic entry trajectory (see Fig. 3) or for trajectories of other payloads, Reynolds numbers on the order of 30 million per foot are developed (10 kft/sec at 30-kft altitude or 15 kft/sec at 40-kft altitude). Comparable magnitude of the Reynolds number may be available only in impulse-type wind tunnels, at much smaller actual flow velocities, i.e., with lower heating rates.

As regards air-breathing propulsion (hypersonic ramjets), because of a high degree of aerodynamic coupling of engine components (intake, combustor, and exhaust nozzle), it has been most difficult to achieve realistic test conditions in conventional facilities. These could be provided by the facility here proposed; moreover as already mentioned, the mechanical guidance present in the rocket acceleration phase could be continued, with small aerodynamic interference, in the hypersonic ramjet test phase (e.g., by three slender, radial supports located along the length of the test tank).

TABLE V
SINGLE-STAGE ROCKET PERFORMANCE

X, kft	5				10				
D, in.	2	4	4	8	24		48		
r, in./sec	20	50	20	50	20	50	20	50	
W _L , Ib	65	16	260	650	130	325	520	1300	
W, lb	910	2275	3640	9100	1820	4550	7280	18, 200	

In all cases: R = 4.73, $\Lambda = 0.85$, $\alpha = 14$, $\gamma = 0.06$ lb/in.³, Q = 0.46, I = 200 sec. V = 10 kft/sec or I = 300 sec. V = 15 kft/sec

From Table I it is apparent that with I = 200 sec and $\Lambda < 0.85$, a velocity of 10 kft/sec is attainable with single stage propulsion. In Table V some typical single-stage data are given for two equivalent conditions: V = 10 kft/sec with I = 200 sec and V = 15 kft/sec with I = 300 sec. Values of $\Lambda = 0.85$ and $\gamma = 0.06$ lb/in. have been taken. With a 5-kft-long, 2-ft-diam launcher, a payload of 65 lb is possible (taking a conservative equivalent burning rate r of 20 in./sec); this increases to 260 lb for a 4-ft-diam launcher. Thus, in the lower hypersonic range, significant performance is possible with a single stage. Moreover, aerodynamic scale can be further increased by pressurization of the test tank.

Higher velocity can be, of course, attained at the cost of multistaging. For example, with two stages (Table IV, a velocity of 14 kft/sec is reached with a 93.5-lb payload in a total propulsive length of only 1512 ft (with I = 200 sec).

Again, it is pertinent to compare the single-stage rocket and gun performances. At 10 kft/sec the maximum gun-launched weight for a 16-in. gun is about 220 lb (see Fig. 4a), i.e., in the order indicated for single-stage rocket propulsion (260 lb for a 48-in.-diam, 5-kft-long launcher, I = 200 sec). The major difference lies, as before, in loads experienced during launching. For the above 260-lb payload case, the rocket thrust pressure is only 240 psi. For a 16-in., 75-caliber-long gun (100-ft barrel), a constant base pressure of about 17 kpsi is required to accelerate a 220-lb payload to 10 kft/sec, but because of imperfections of the actual launch cycle, a peak pressure at least twice as large would be estimated. Thus, although the gun offers the velocity performance at the 10-kft/sec level, it subjects the payloads to base pressures high enough to pose structural and instrumentation problems.

SECTION VI CONCLUSIONS

Results presented here indicate that the proposed facility would be capable of testing much larger models than is now possible in the velocity range of major interest: in the examples considered, model/sabot weights range from 10 to 1 lb at velocities from 25 to 35 kft/sec, respectively.

In the admittedly highly idealized cases discussed, facilities of the proposed type appear rather modest in overall size (except for length). Their main attractiveness lies in performance higher than can be now envisaged with other types of ground simulation devices, and no obvious physical, as opposed to economic, limitations on future performance, i.e., high performance potential. Moreover, through decoupling of velocity and atmosphere duplication, such facilities offer complete freedom of environmental simulation. Although a full-scale (in terms of flight hardware) facility could not be in general considered practical, the relatively large model scale offered by the proposed facility would allow a significant range of scaling experiments to be performed. In view of relatively small launch accelerations, telemetry of model data (subject to model flow field interference) would be practical. Coupled with external range observations, the variety and quality

of experimental data could attain standards comparable to those of the conventional wind tunnel test technique.

As part of this study, the use of a high-speed gun as the initial propulsion stage was investigated. It was found that, except for the lower velocities (in the 25-kft/sec range), multistage rocket propulsion is required in conjunction with the gun to cover the velocities and payloads of interest. Thus, since the simplicity of single-stage rocket operation is lost, and the multistage rocket and launcher design must be developed, it seems desirable to favor pure, multistage rocket systems, which, moreover, are free from high gun-launch accelerations.

The mechanical design of the proposed facility is not considered in this paper, the purpose being to establish the fundamental feasibility related to propulsive performance. The motion of rocket stages in long tubes at very high speeds, and the staging technique are problems which will undoubtedly require much developmental work. Compared to the rocket sled shoe situation, Ref. 22, the high speed motion of rocket stages in evacuated tubes is quite different and probably less critical. The fact that no difficulties have been encountered with launching of cylindrical plastic projectiles by light gas guns at velocities in the 25 to 35 kft/sec range is encouraging in this connection.

REFERENCES

- 1. "An Assessment of Our Present Status and Future Requirements for High Temperature Hypersonic Facilities." Round Table Discussion, The High Temperature Aspects of Hypersonic Flow, Pergamon Press, 1964. ("What are the limitations of existing facilities?", contribution by J. Lukasiewicz, pp. 759-768)
- 2. Lukasiewicz, J. "Constant Acceleration Flows and Applications to High Speed Guns." AEDC-TR-66-181 (AD642333), 1966, and AIAA Journal, Vol. 5, No. 11, November 1967, p. 1955.
- 3. Whitfield, J. D. and Potter, J. L. "Simulating High-Speed Aerodynamics." Space/Aeronautics, March 1967.
- 4. Poisson-Quinton, Philippe. "From Wind Tunnel to Flight." 30th Wright Brothers Lecture, AIAA Journal of Aircraft, Vol. 5, No. 3, May June 1968, p. 193.
- 5. Brahinsky, H. and Northcutt, D. "Mollier Diagram for Equilibrium Air." VKF, ARO, Inc., AEDC, February 1967.
- 6. Hall, J. Gordon and Treanor, Charles E. "Nonequilibrium Effects in Supersonic Nozzle Flows." Cornell Aeronautical Laboratory Report, CAL-163, March 1968, and AGARDograph No. 124.
- 7. Norman, W. S. and Chmielewski, G. E. "Limit Duplication Lines for Isothermal, Constant Loading Factor, JxB Accelerators." AEDC-TR-65-37 (AD457134), February 1965.
- 8. Rose, P. H. and Stankevics, J. O. "Stagnation-Point Heat-Transfer Measurements in Partially Ionized Air." AIAA Journal, Vol. I, December 1963, pp. 2752-2763.
- 9. Gruszcynski, J. S. and Warren, W. R. "Experimental Heat-Transfer Studies of Hypervelocity Flight in Planetary Atmospheres." <u>AIAA Journal</u>, Vol. II, September 1964, pp. 1542-1550.
- 10. Trimpi, Robert L. "A Preliminary Theoretical Study of the Expansion Tube, A New Device for Producing High-Enthalpy Short-Duration Hypersonic Gas Flow." NASA-TR-R-133, 1962.
- 11. Trimpi, Robert L. and Callis, Linwood B. "A Perfect Gas Analysis of the Expansion Tunnel, A Modification to the Expansion Tube." NASA-TR-R-223, April 1965.

- 12. Stoddard, F., Hertzberg, A., and Hall, J. Gordon. "The Isentropic Compression Tube: A New Approach to Generating Hypervelocity Test Flows with Low Dissociation." Proceedings, Fourth Hypervelocity Techniques Symposium, Arnold Engineering Development Center, November 1965.
- Jones, J. J. "Some Performance Characteristics of the LRC-3-3/4-Inch Pilot Expansion Tube Using an Unheated Hydrogen Driver." Givens, John J., Page, William A., and Reynolds, Robert M. "Evaluation of Flow Properties in a Combustion-Driven Expansion Tube Operating at 7.5 km/sec." Norfleet, Glenn D., Lacey, John J., Jr., and Whitfield, Jack D. "Results of an Experimental Investigation of the Performance of an Expansion Tube." Spurk, Joseph H. "Design, Operation and Preliminary Results of the BRL Expansion Tube." Proceedings, Fourth Hypervelocity Techniques Symposium, Arnold Engineering Development Center, November 1965.
- 14. Clemens, P. L. and Kingery, M. K. "Development of Instrumentation for a Hypervelocity Range." Proceedings, First Hypervelocity Techniques Symposium, Denver, Colorado, October 20-21, 1960.
- 15. Seiff, Alvin. "Ames Hypervelocity Free-Flight Research."

 Astronautics and Aerospace Engineering, Vol. 1, No. 11,

 December 1963, pp. 16-23.
- 16. Greene, Gerald L. and Wood, Gordon E. "A High Performance Five Inch Diameter Rocket Motor." Atlantic Research Corporation, Alexandria, Va., ISTRACON Structures Working Group, August, 1966. Also private communications from R. C. Maydew and L. V. Feltz, Sandia Corporation, Albuquerque, N. M.
- 17. Bull, G. V., Lyster, D., and Parkinson, G. V. "Orbital and High Altitude Probing Potential of Gun Launched Rockets." McGill Univ., Space Research Institute, Report R-SRI-H-R-13, October 1966. Also private communication for G. V. Bull, July 1967.
- 18. Vest, D. C. "An Experimental Travelling Charge Gun." Ballistic Research Laboratories Report 773, October 1951.
- 19. Vinti, J. P. "Theory of the Rapid Burning of Propellants. Ballistic Research Laboratories Report 841, December 1952.
- 20. Baer, P. G. "The Travelling Charge Gun as a Hypervelocity Launching Device." Fourth Hypervelocity Impact Symposium, APGC-TR-60-39 (III), September 1960.

- 21. Plattner, C. M. "Liquid-Filled Cases Stretch Rocket Range."

 Aviation Week and Space Technology, March 25, 1968,
 p. 57-61.
- 22. Rigali, D. J. and Feltz, L. V. "The Application of High-Speed Monorail Rocket Sleds to Aerodynamic Testing at High Reynolds Numbers." AIAA 3rd Aerodynamic Testing Conference, San Francisco, April, 1968, AIAA Paper No. 68-365.

APPENDIX ROCKET PROPULSION PERFORMANCE PARAMETERS

In absence of gravity and drag, the motion of a single-stage rocket is described by

$$F = \dot{W}_{n}I = (W/g) dV/dt \qquad (I-1)$$

where

F = thrust

 \dot{W}_{D} = propellant weight flow rate

I = specific impulse

W = total rocket weight

V = rocket velocity

Assuming F, \dot{W}_{p} , and I invariant with time, and since

$$\dot{W}_{\rm p} = -dW/dt \qquad (I-2)$$

Eqs. (I-1) and (I-2) give

 $dV = -gI \ dW/W$

and

$$V_1 = gI \ln R_1 \tag{I-3}$$

where V = 0 when W = W_1 , and velocity V_1 is attained when W = W_1 - W_{p_1} , with R_1 = $W_1/(W_1-W_{p_1})$ and W_{p_1} = weight of expended propellant.

The corresponding duration of motion is

$$t_{1} = W_{p_{1}}/\dot{W}_{p}$$

$$= \frac{W_{1}I}{F} (1 - 1/R_{1})$$

$$= \frac{(W_{1} - W_{p_{1}})I}{F} (R_{1} - 1)$$
(I-4)

The distance required to attain velocity V_1 is obtained from Eq. (I-3) and

$$dX = V dt = gI \ln \frac{W_1}{W_1 - \dot{W}_n t} dt$$

Integrating, with V = 0, $W = W_1$ at t = 0 and $V = V_1$, $W = W_1 - W_{p_1}$ at time t_1 ,

$$X_1 = g I \left(\frac{W_1 - W_{p_1}}{\psi_p} \ln \frac{1}{R_1} + t_1 \right)$$

or, using Eq. (I-4),

$$\frac{X_1F}{(W_1 - W_{p_1})_g I^2} = R_1 - 1 - \ln R_1$$
 (I-5)

In general, if $V = V_S$ at t = 0, Eqs. (I-3) and (I-5) read

$$V_1 = V_g + g I \ln R_1 \qquad (I-6)$$

and

$$\frac{X_1F}{(W_1 - W_{p_1}) g I^2} = \frac{V_s}{gI} (R_1 - 1) + (R_1 - 1 - \ln R_1)$$
 (I-7)

$$= R_1 \left(\frac{V_s}{gI} + 1 \right) \text{ for } R_1 >> 1$$

The above equations hold for a single stage of rocket propulsion. Because the mass ratio R and the specific impulse I attainable in practice are limited, multistage rocket propulsion must be resorted to in order to reach orbital and superorbital velocities.

The treatment of ideal performance of multistage rockets follows that given by Geckler.* A multistage rocket is characterized by:

N = total number of stages of propulsion

$$R_i = W_i/(W_i - W_{pi}) = stage mass ratio$$

with W_i = initial rocket weight (all stages higher than and including i)

 W_{p_i} = weight of propellant used in the i-th stage of propulsion

$$\Lambda_i = \frac{\mathbf{w}_{\mathbf{p}_i}}{\mathbf{w}_{\mathbf{p}_i} + \mathbf{w}_{\mathbf{e}_i}} = \text{motor propellant fraction}$$

^{*}R. D. Geckler, "Ideal Performance of Multistage Rockets," ARS Journal, June 1960, pp. 531-536.

with W_{e_i} = empty weight of the i-th stage

$$a_i = \frac{W_i}{W_{i+1}} = \text{stage payload ratio}$$

It follows that

$$a_i = \frac{\Lambda_i}{\Lambda_i - 1 + 1/R_i} \tag{I-8}$$

It is evident that stage payload vanishes unless

$$\Lambda_i > 1 - 1/R_i = \Lambda_{m_i} \tag{I-9}$$

where Λ_{m_i} is the minimum permissible motor propellant fraction.

Function (I-8) is plotted in Fig. I-1 for several Λ = constant values.

From Eq. (I-3), the velocity increment Δv_i due to the i-th stage of propulsion is

$$\Delta v_i = \ln R_i \tag{I-10}$$

where v = V/(gI).

Using (I-9), the velocity increment corresponding to the maximum permissible motor propellant fraction Λ_{m_i} is given by

$$\Delta v_{m_i} = -\ln(1 - \Lambda_{m_i}) \qquad (I-11)$$

In the ideal, optimum case where R_i , Λ_i , and α_i are each constants, unchanged from stage to stage, we have, dropping the i-subscripts, for N-stage propulsion from zero velocity ($V_S = 0$),

$$v_o = N\Delta v = N \ln R = \ln R_o \qquad (I-12)$$

where $R_0 = R^N$ = effective mass ratio (I-13)

$$\alpha = \frac{\Lambda}{\Delta - 1 + 1/R}, \qquad (I-14)$$

$$\Lambda_{\rm m} = 1 - 1/R \tag{I-15}$$

From definition of α_i it follows that

$$W_{i}/W_{N+1} = W_{i}/W_{L} = \alpha^{N-(i-1)}$$
 (I-16)

and

$$\alpha_{o} = W_{1}/W_{N+1} = W_{1}/W_{L} = \alpha^{N}$$
 (I-17)

with W_L = weight of payload stage.

Noting that, according to (I-12), the velocity increment due to each stage is $\Delta v = v_0/N$, the distance X_i required for the i-th stage propulsion is given by

$$Q_i = C_i X_i = \frac{1}{R} \{ (R-1) + [(i-1) (R-1) - 1] \ln R \}$$
 (I-18)

where

$$C_i = F_i/(gI^2W_i)$$

and F_i is the thrust of the i-th stage (see also Eq. (I-22)). Function (I-18) is plotted in Fig. I-2 for values of i from 1 to 6.

The total distance X_0 necessary to reach velocity $v_0 = N\Delta v$ is obtained from Eq. (I-18) as $X_0 = \sum_{i=1}^{N} X_i$.

Assuming C_i = C = constant for all stages,

$$Q_o = CX_o = \frac{N}{R} \{ (R-1) + [(R-1) (N-1)/2 - 1] \ln R \}$$
 (I-19)

and, from Eqs. (I-17) and (I-18),

$$\frac{X_i}{X_0} = \frac{(R-1) + [(i-1)(R-1) - 1] \ln R}{N \{ (R-1) + [(R-1)(N-1)/2 - 1] \ln R \}}$$
 (I-20)

Function (I-19) is plotted in Fig. I-3 for values of N from 1 to 6.

Thus, the relative stage propulsion distance X_i/X_0 is only a function of i, R, and N (provided C_i = constant).

From the above analysis the thrust and distance required to attain a specified velocity with a payload WL, for a given number N of propulsive stages, characterized by a certain motor propellant fraction A and with a propellant of a specific impulse I, can be determined. The practical problem concerns, of course, the design of a rocket system with which the performance, as specified above, can be obtained. Here, we shall confine ourselves only to a general consideration of the rocket motor design parameters.

The rate of propellant flow \dot{W}_p can be written as

$$\tilde{W}_{p} = r A \gamma \qquad (I-21)$$

where

r = equivalent propellant burning velocity

A = cross-sectional area of rocket motor

y = specific weight of propellant

From Eqs. (I-1) and (I-21), the coefficient C_i (see Eq. (I-18) is expressed as

$$C_{i} = \frac{r_{i} A_{i} \gamma}{g I W_{i}}$$

and therefore

$$\frac{r_i A_i \gamma X_i}{g I W_i} = f(i, N, R) \qquad (I-22)$$

Also, $A_i = \pi D_i^2/4$ with $D_i = \text{rocket motor diameter}$.

The acceleration "a" (in g-units) resulting from rocket propulsion is given by

$$a'_{i} = F_{i}/W_{i}$$
 (minimum acceleration, at the beginning of i-th stage propulsion). (I-23)

and

$$a_i'' = F_i/(W_i - W_{p_i})$$

$$= R_i F_i/W_i$$

$$= R_i a_i^i \qquad \text{(maximum acceleration, at termination of i-th stage propulsion)} \qquad (I-24)$$

In terms of design parameters,

$$a_i^{"} = \frac{R_i r_i A_i \gamma I}{W_i \alpha^{N-(i-1)}}$$
 (I-25)

With I = constant and C = constant, see Eq. (I-18), accelerations a^{\prime} and $a^{\prime\prime}$ are constant for all stages of propulsion. Thus,

$$a'' = R r \gamma I (A/W)_{i}$$
 (I-26)

Summarizing, the calculation of an N-stage rocket system, using a propellant of specific impulse I, and required to accelerate a given payload W_L to velocity V_O involves the following steps:

- (i) Stage mass ratio R is defined by Eq. (I-12) in terms V_O and N;
- (ii) For a specified motor propellant fraction Λ , the stage payload ratio α is given by Eq. (I-14), and hence the weight of all stages is given in terms of payload W_{L} ;
- (IIi) Parameters Q_i and Q_o are given in terms of (i, R, N), Eqs. (I-18) and (I-19). From Eq. (I-22) it is evident that variables r_i , A_i , W_i , and X_i (i.e., W_L) may take any values provided f (i, N, R) = constant. Therefore, Eq. (I-22) defines the available design tradeoffs.

Parameters, A, r, and A relate directly to rocket motor design, which is not considered in this report. Rather, the area A is identified with the rocket motor or stage cross-sectional area, and r is taken as the equivalent burning velocity (based on A and not on the actual surface of propellant combustion). Various designs of propellant grain are then reflected in the (equivalent) values of r, which may be several times higher than the actual burning speed.

As regards combustion chamber pressure and nozzle expansion ratio, approximate values can be obtained from the definition of thrust coefficient $C_{\mathbf{F}}$ given by

$$C_{F} = \frac{F}{A^* p_{c}} \qquad (I-27)$$

where A^* = nozzle throat cross-sectional area and p_C = chamber pressure.

With A/A* =
$$\epsilon$$
 and using Eq. (I-21)
$$C_F = I \gamma \epsilon r/p_c \qquad (I-28)$$

For a given propellant, thrust coefficient is mainly a function of p_C and ϵ , and may take values between 1 and 2; a value of 1.5 may be taken as representative and expansion ratio ϵ obtained as a function of r and p_C .

As regards rocket propulsion systems considered in this report, it is of interest to evaluate the practical maximum values for the equivalent propellant burning velocity r, as determined by propellant properties rather than motor design. Typically, for $C_F = 1.5$, I = 200 sec, $\gamma = 0.06$ lb/in.³, $p_C = 2000$ psi and assuming a minimum nozzle expansion ratio $\epsilon = 2$, Eq. (I-28) gives $r_{max} = 125$ in./sec.

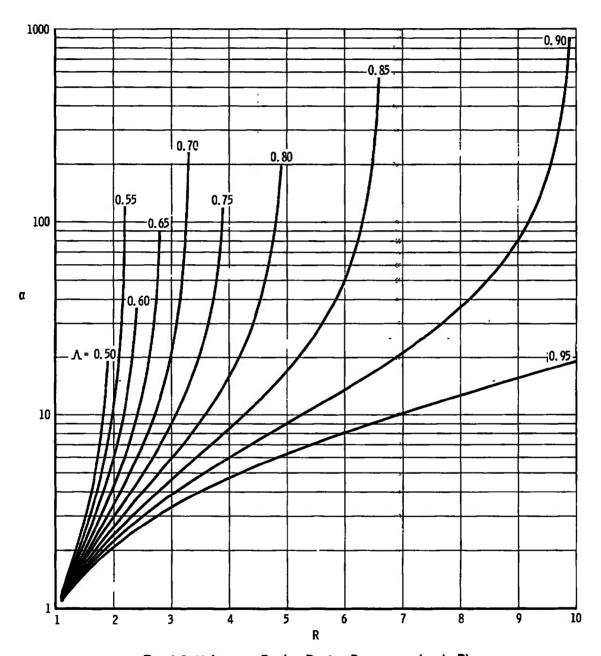


Fig. I-1 Multistage Rocket Design Parameters (α , Λ , R)



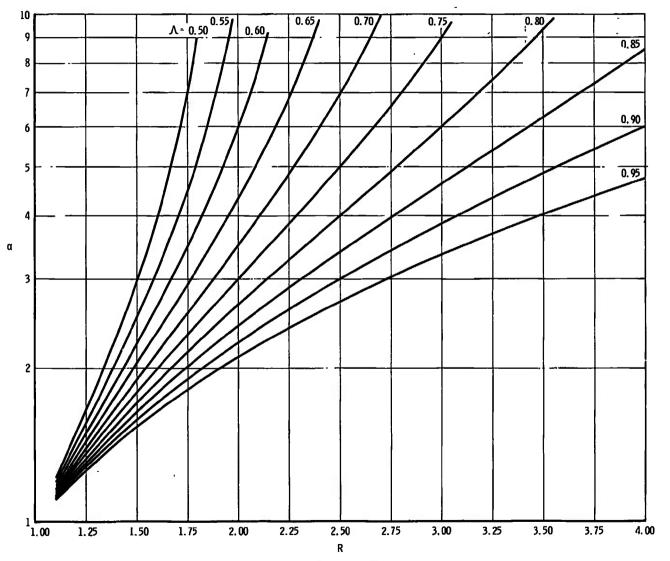


Fig. I-1 Continued

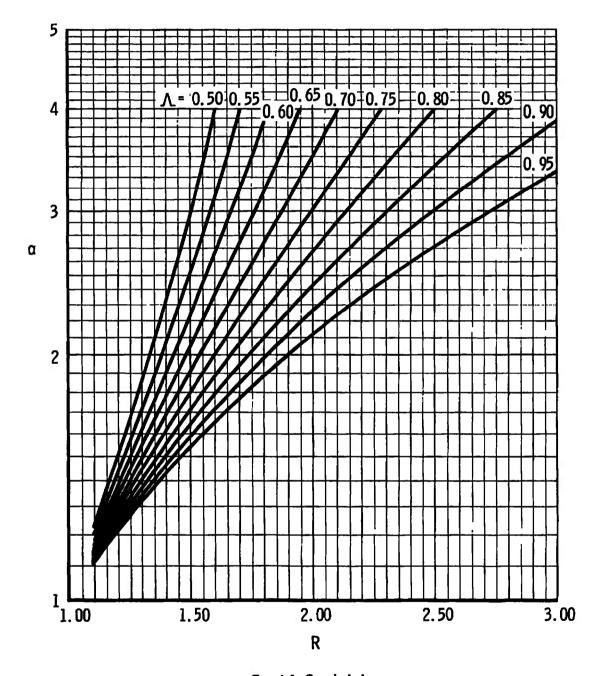


Fig. I-1 Concluded

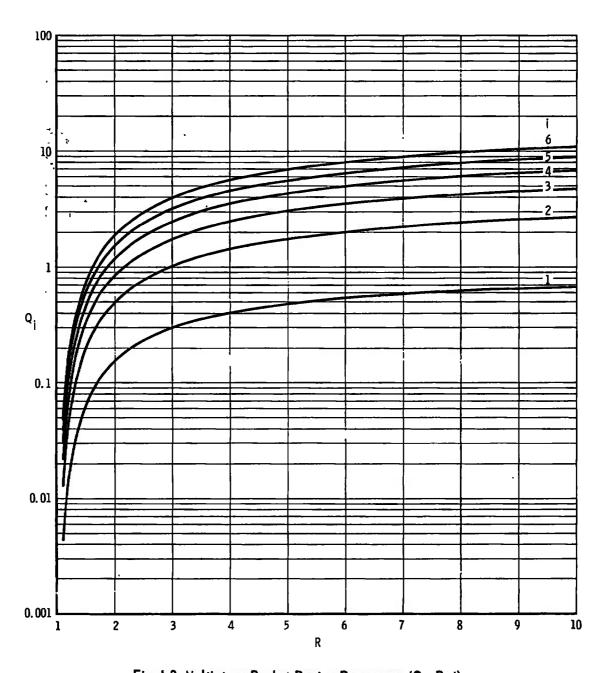


Fig. 1-2 Multistage Rocket Design Parameters ($\mathbf{Q_i}$, R, i)

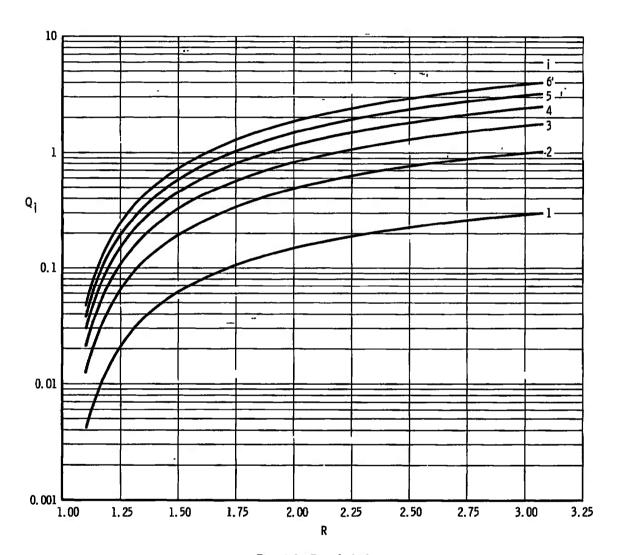


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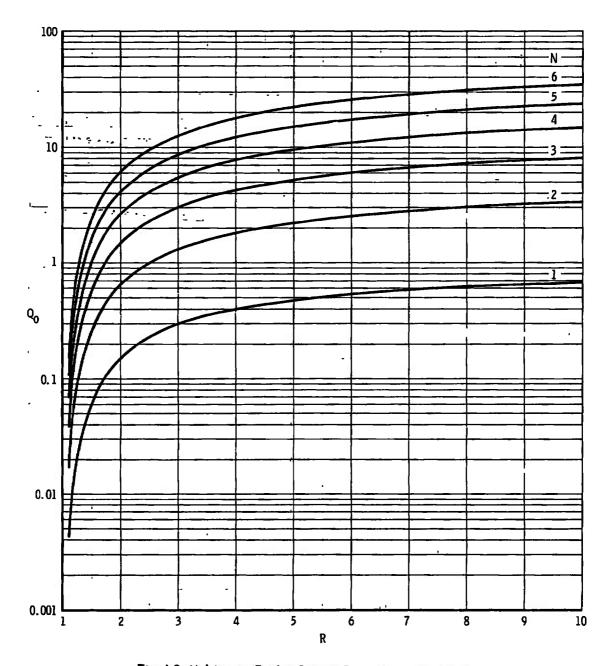


Fig. 1-3 Multistage Rocket Design Parameters ($\mathbf{Q_o}$, R, N)

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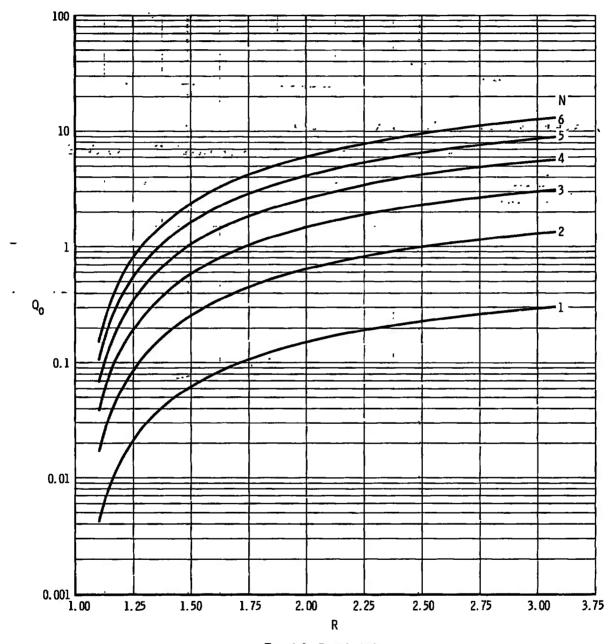


Fig. 1-3 Concluded

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stability, and drag studies. It is estimated that model/sabot packages

kft/sec, respectively. The proposed test technique as applied to lower hypersonic speeds, to the duplication of high Reynolds numbers, and to

weighing from 10 to 1 1b could be launched at speeds from 25 to 35

hypersonic ramjet testing is discussed.

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